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Anomalous Creation of Branes

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Abstract: In certain circumstances when two branes pass through each other a third brane is produced stretching between them. We explain this phenomenon by the use of chains of dualities and the inflow of charge that is required for the absence of chiral gauge anomalies when pairs of D-branes intersect.

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At large distances D-branes [1, 2] interact through the long-range fields of supergravity. At sub-stringy scales, on the other hand, their interaction is more usefully described in terms of the creation and subsequent annihilation of (virtual) pairs of open strings stretching between the branes. Such pairs may materialize from the vacuum [3], a phenomenon analogous to pair creation in an electric field. There are however special situations, to be discussed here, in which *a single* open string *must* materialize when two D-branes cross. Hanany and Witten [4] pointed out a similar phenomenon, in which a stretched D3-brane is produced when a D5-brane and a NS 5-brane cross. In this note we will explain how such phenomena can be related by chains of dualities to the (abelian) anomaly equation describing charge inflow [5] on the intersection of branes [6].

A set of m coincident type-II D-branes is described by a supersymmetric field theory with gauge group $U(m)$. When two such sets of D-branes intersect, the field content in the intersection domain includes the pull-backs of the corresponding $U(m)$ and $\tilde{U}(\tilde{m})$ gauge potentials, as well as the pull-back of the bulk graviton field. There are circumstances in which this effective field theory is chiral [6]. For this to arise the two sets of coincident branes must intersect in such a manner that the space-time dimension of the intersection domain is 2 mod 4, and there are no spatial directions transverse to both sets. Two distinct configurations of intersecting type-II D-branes that result in chiral supersymmetry in the intersection region are: *(a)* two D5-branes of type-IIB theory intersecting on a string, or *(b)* two D7-branes of type-IIB theory intersecting on a 5-brane. Other chiral configurations in either type IIA or IIB are obtained from these by T-dualizing coordinates transverse to the intersection region. Let us denote the world-volume actions of the two individual branes and of the intersection region by $\mathcal{S}$, $\tilde{\mathcal{S}}$ and $\mathcal{I}$, respectively. Under a gauge transformation, the variation of $\mathcal{S}$ and $\tilde{\mathcal{S}}$ has a (boundary) piece localized at the intersection, which precisely cancels the anomalous variation of $\mathcal{I}$ [6]. Physically, since the embedding theory as well as the theory on either of the two D-branes is non-anomalous, the apparent charge violation in the presence of background fields in the intersection region has to be accounted for by inflow of charge from the branes. This is an application of the 'anomaly inflow' argument of [5].

Let us focus first on configuration (a) with two intersecting D5-branes lying in the planes (12345) and (16789), respectively. We will take the direction X_1 to be a circle with radius L_1. Furthermore, it will be sufficient to consider the abelian case, in which there is an independent maximally-supersymmetric $U(1)$ vector multiplet living in each brane. The spectrum in the intersection region consists of the pull-backs of these two abelian gauge potentials, together with a single Weyl fermion with $U(1) \times \tilde{U}(1)$ charge $(+,-)$. This fermion, the lightest state of an oriented open string stretching between the two intersecting branes, does not transform under $(0,8)$ supersymmetry. Now consider switching on time-dependent Wilson lines $A_1(t)$ and $\tilde{A}_1(t)$ on the two branes. The anomaly equation implies that the net inflow of fermion number onto the intersection domain is

$$\Delta N = L_1 \int dt \, \left(\frac{dA_1}{dt} - \frac{d\tilde{A}_1}{dt} \right) . \qquad (1)$$

A T-duality along the first dimension transforms the D5-branes into D4-branes parallel to the hyperplanes (2345) and (6789). Furthermore the gauge fields $2\pi\alpha' A_1$ and $2\pi\alpha' \tilde{A}_1$ become the transverse coordinates X_1 and $\tilde{X}_1$ of the 4-branes, while the radius L_1 gets mapped to $L_1' = \alpha'/L_1$. The anomaly equation thus takes the form

$$\Delta N = \frac{1}{2\pi L_1'} \int dt \left(\frac{dX_1}{dt} - \frac{d\tilde{X}_1}{dt} \right) , \tag{2}$$

where the right-hand side may now be recognized as the net (forward minus backward) number of times the two D4-branes cross while moving on the circle of radius L_1'. This phenomenon can be understood as follows: the energy levels of a ground-state (chiral fermionic) open string stretching between the two branes are

$$E_n = \frac{1}{2\pi\alpha'}(X_1 - \tilde{X}_1 + 2\pi n L_1'), \tag{3}$$

where n is the number of times the string winds around the circle. Every time the two D4-branes pass through each other, an energy level crosses the zero axis creating a single particle or hole. This is precisely the content of the anomaly equation, as illustrated in figure 1.

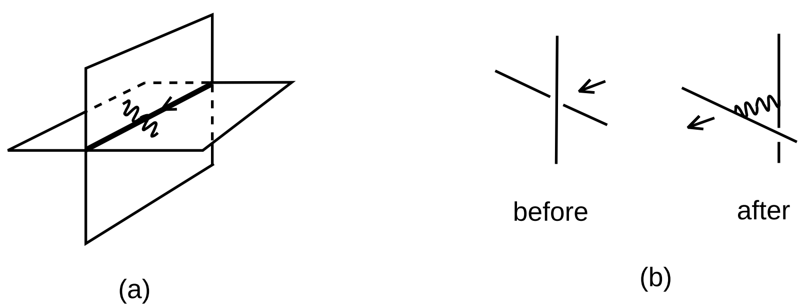

Figure 1: (a) Two five-branes intersecting in a single compact direction. Turning on an electric field along the circle induces an electric current due to the flow of open strings joining the branes. (b) After T-duality this describes two D4-branes that have a relative velocity along the circle and an open string is created each time the branes pass through each other.

By using chains of dualities this process can be transformed into more exotic phenomena. We will denote a process in which two branes, A and B, create a stretched brane C when they pass through each other by $A \otimes B \hookrightarrow C$. The basic example, the creation of a stretched fundamental string when two mutually transverse D4-branes cross, reads in this notation: $D(2345) \otimes D(6789) \hookrightarrow F(1)$. The parentheses indicate the hyperplanes along which the various branes lie. Consider now the following sequence of dualities,

$$\begin{array}{ll}
(\mathrm{IIA}) & D(2345) \otimes D(6789) \hookrightarrow F(1) \\
 & \qquad\qquad \Big\downarrow T(6) \\
(\mathrm{IIB}) & D(23456) \otimes D(789) \hookrightarrow F(1) \\
 & \qquad\qquad \Big\downarrow S \\
(\mathrm{IIB}) & NS(23456) \otimes D(789) \hookrightarrow D(1) \\
 & \qquad\qquad \Big\downarrow T(56) \\
(\mathrm{IIB}) & NS(23456) \otimes D(56789) \hookrightarrow D(156)
\end{array}$$

Here $T(ij...)$ is a T-duality along the indicated dimensions, while S is the strong-weak coupling duality, which exchanges the Neveu-Schwarz (NS) and Dirichlet fivebranes of type IIB, leaves the D3-brane invariant, and transforms the fundamental string to a D-string. What we learn from this chain of dualities is that a D3-brane must be created when a NS-brane and a D5-brane, sharing two common dimensions, pass through each other. This is precisely the phenomenon observed by Hanany and Witten [4].

Another sequence of dualities gives

$$\begin{array}{ll}
(\mathrm{IIA}) & D(2345) \otimes D(6789) \hookrightarrow F(1) \\
 & \qquad\qquad \Big\downarrow T(67) \\
(\mathrm{IIA}) & D(234567) \otimes D(89) \hookrightarrow F(1) \\
 & \qquad\qquad \Big\downarrow S \\
(\mathcal{M}) & KK(234567) \otimes M(89) \hookrightarrow M(1\,10)
\end{array}$$

where S is now the duality that lifts type-IIA theory to $\mathcal{M}$-theory compactified on a circle. $\mathcal{S}$ transforms the D6-brane to the Kaluza-Klein monopole and lifts the D2-brane to the M-brane transverse to the tenth dimension. It also lifts the fundamental string to the *wrapped* M-brane. We conclude that a wrapped M-brane must be created whenever an unwrapped one passes through a transverse Kaluza-Klein D6-brane. Alternatively, the original pair of crossing D4-branes can be lifted directly to $\mathcal{M}$-theory where the process corresponds to the creation of a wrapped membrane whenever two wrapped but otherwise transverse fivebranes cross each other.

Our discussion so far has been based on the gauge anomaly equation in two dimensions, which we have interpreted in terms of the net number of times two D4-branes cross when moving along their unique common transverse direction. The six-dimensional anomaly equation in the overlap domain of two (sets of) D7-branes (case (b) above) can be given a similar interpretation. The generic situation consists of m $D(1234567)$ and $\tilde{m}$ $D(1234589)$ branes, intersecting on some five-manifold $\mathcal{M}^5$ which spans the dimensions (12345). The net inflow of fermions on this intersection domain reads [6]

$$\Delta N = \int_{R\times\mathcal{M}^5} \mathrm{tr}_m \exp\left(\frac{\mathcal{F}}{2\pi}\right) \ \mathrm{tr}_{\tilde{m}^*} \exp\left(\frac{\tilde{\mathcal{F}}}{2\pi}\right) \ \hat{A}(\mathcal{R}) \ \Bigg|_{6-\mathrm{form}} , \tag{4}$$

where $\mathcal{F}$ and $\tilde{\mathcal{F}}$ are the (pull backs) of the gauge field strengths and $\mathcal{R}$ is the (pull back) of the curvature. The traces are in the fundamental (conjugate) representation of $U(m)$ (respectively $\tilde{U}(\tilde{m})$), and multiplication is in the sense of forms. The roof genus has an expansion in forms, $\hat{A} = 1 - p_1(\mathcal{R})/24 + \cdots$, where p_1 is the first Pontryagin class and $\cdots$ represents higher-rank $4n$-forms.

We shall consider the particular example in which the intersection domain is the orthogonal torus, $\mathcal{M}^5 = (S^1)^5$, where each of the five circles has radius L_i $(i = 1, \cdots, 5)$. It will also be sufficient to consider m coincident seven-branes intersecting a single seven-brane ($\tilde{m} = 1$) in a five-dimensional region, and to switch off the $U(1)$ gauge field on the latter. The anomalous $U(1)$ charge is induced by first turning on a time-dependent Wilson line, $A_1(t)$, along the x_1 direction in the abelian $U(1)$ factor of the $U(m)$ gauge group, and taking all other fields to be independent of t and x_1. A T(1) duality transformation maps this configuration to a bunch of coincident six-branes, $D(234567)$ moving around the dual circle of radius L_1' and passing through a single six-brane, $D(234589)$, that is stationary. The branes intersect instantaneously on the four-torus $(S^1)^4$ that spans the directions (2345) and the anomaly equation takes the form

$$\Delta N = \frac{1}{2\pi L_1'} \int dt \frac{dX_1}{dt} \times \int_{(S^1)^4} \frac{1}{4\pi^2} \mathrm{tr}_m \mathcal{F}\wedge\mathcal{F}, \tag{5}$$

where X_1 is the position on the (dual) circle of the m coincident moving branes. The net charge inflow is thus given by

$$\Delta N = n_c k \ , \tag{6}$$

where n_c is the net number of times the two sets of branes encounter each other and k is the instanton number of the $U(m)$ gauge fields living on the moving branes and pulled back to the four-torus $(S^1)^4$.

We may interpret equation (6) in terms of the basic phenomenon of crossing D4-branes, described previously in figure 1. Indeed, in the limit where the k gauge instantons have (almost) zero size, they are equivalent to k two-branes, $D(67)$, bound to the m coincident moving six-branes [7, 8]. As these D2-branes pass through the stationary D6-brane they each create one fermionic string, a process T-dual to the process of figure 1. An alternative gauge configuration consists of smooth Wilson lines in the $U(1)$ factor of the $U(m)$ gauge group,

$$A_2 = \frac{n_2 x_4}{2\pi L_2 L_4}, \qquad A_3 = \frac{n_3 x_5}{2\pi L_3 L_5}. \tag{7}$$

Here n_2 and n_3 are integers, consistently with the quantization of magnetic flux, and the total instanton number is $k = n_2 n_3 m$. A T(23) duality transformation maps all D6-branes to D4-branes. The stationary brane now spans the hyperplane (4589) while all m moving branes are uniformly rotated away from the hyperplane (4567) since the Wilson lines become the transverse displacements

$$X_2 = n_2 \frac{L_2'}{L_4} x_4, \qquad X_3 = n_3 \frac{L_3'}{L_5} x_5 \ . \tag{8}$$

One may visualize the rotated branes as spiralling around the (dual) torus (234567). Each such spiralling D4-brane intersects the stationary D4-brane in $n_2 n_3$ points as it passes through, in accordance again with the anomaly equation (6) (see figure 2).

The case $\mathcal{M}^5 = S^1 \times K3$ can be analyzed similarly. The T(1) duality transformation, which sets the branes in motion, must now be followed by a mirror map which acts as T-duality on the fiber tori of the $K3$ surface [9]. The end result is (sets of) D4-branes, which intersect the $K3$ surface on two-cycles and intersect each other at points as they cross. The mirror map from these two-cycles to marked points and/or the entire surface is subtle, however, because the latter carries a negative unit of induced point-charge [10].

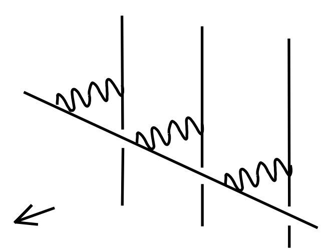

Figure 2: Two sets of D4-branes, arising by T-duality from intersecting D7-branes, produce a number of strings equal to the total number of intersection points as they move past each other. The horizontal line is the stationary brane, whereas the vertical lines represent either (a) k zero-size instantons, or (b) k segments of the m (continuous) spiralling and moving branes, discussed in the text.

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